Using Fractional Gaussian Noise Models in Orbit Determination

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HERE is a re-statement of Theorem 4 in "Using Fractional Gaussian Noise Models in Orbit Determination," published in the *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 4, pages 593–607, using a generalization that broadens functions $\{F_i : i = 1, \ldots, n\}$ to a wider variety of possible transformations. In the published paper it must be one-to-one and onto for finite partitions of sets of nonzero probability. Now, the transformation need only be one-to-one and onto for finite partitions of sets that have nonzero probability and whose random variables are statistically independent. Also, the description of the partitioning in the first sentence of the proof is clarified here.

Theorem 4: Let $\{\boldsymbol{X}_i : i = 1, \ldots, n\}$ be an ordered set of random vectors and operator $\boldsymbol{T} \equiv (T_1, \ldots, T_n)$ be a transformation for which vectors $\boldsymbol{y}_i = T_i(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n) = F_i(\boldsymbol{x}_i)$ for some function F_i. Define $S_i \ni \forall \boldsymbol{x}_i \in S_i$, $p_{X_1, \ldots, X_n}(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n) = p_{X_1}(\boldsymbol{x}_1) \ldots p_{X_n}(\boldsymbol{x}_n)$ and $p_{X_i}(\boldsymbol{x}_i) > 0$.

Furthermore, suppose F_i is one-to-one and onto for disjoints sets $S_{ij} \subseteq S_i$; $\bigcup_{j=1}^{k_i} S_{ij} = S_i$.

Then, given image vectors $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_n \ni F_i^{-1}(\boldsymbol{y}_i) \subseteq S_i$

$$p_{Y_1, \ldots, Y_n}(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_n) = p_{Y_1}(\boldsymbol{y}_1) \cdots p_{Y_n}(\boldsymbol{y}_n)$$

The proof proceeds as before, but with the following change: The phrase, "Because $\times_{i \neq j} S_i \times S_j$ partitions $\times_i S_i \ldots$", currently located in the beginning of the first sentence of the proof, should instead read: "Because the sets $\times_{i=1}^{n} S_{ir_i}$, where $r_i = 1, \ldots, k_i$, partition $\times_{i=1}^{n} S_i \ldots$".